

Johann Sebastian

Bach

*Mystic
and
Mathematician*

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November 30, 1992

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Even a brief exposure to the music of Johann Sebastian Bach, (for example, the Two-Part Inventions), will leave the careful listener with a sense of mathematical precision and logical structure. To one extent or another, this has been recognized by composers (and others) for many years. Over time, several scholars and students of Bach have revealed that his music displays all manner of mathematical relationships; some that were apparent only after careful study. This would naturally whet one's curiosity about precisely *which* relationships, and what effect, (if any), these had upon the creative process which led to such magnificent music. Some have concluded that any mathematical patterns which may exist were either: (1) due to coincidence, or (2) due to the inherent mathematical nature of the physics of music and the nature of human ear response. While there may be merit to both of these explanations, it is obvious that there has been much more deliberate manipulation on the part of Bach than had previously been imagined. The quantity and variety of instances of math found by others in Bach's music range from examples of obvious, purposeful inclusion to examples of wild-eyed hypothesis of Gordian complexity, which, if true, would have the effect of granting supernatural powers to the venerable composer.

Other composers before, during, and after Bach's period have employed mathematical patterns or puzzles in their music: Arnold Schoenberg, (whose serial techniques Bach anticipated, to an extent), structured *Pierrot Lunaire* so that the music proceeds to the middle of the piece, then goes backward through the same material to the end. In *Ludus Tonalis*, Paul Hindemith uses the same musical material for the Postlude as for the Prelude, except that in the Postlude, it is played in retrograde inversion. Mendelssohn was familiar with Bach's mirror and crab fugues; his double, triple, and quadruple fugues, and considered them to be "mathematical projections", that could only be understood "by calculation". Haydn was also familiar with these examples, but had no love for them at all, (a rather strange attitude, considering that he himself composed a minuet *al rovesco*, "in reverse"; contained within his Piano Sonata in A). Mozart appreciated the mathematical aspect, perhaps for no other reason than he thought it necessary for a true composer to be familiar with and proficient in all areas of his art (Jacob, 78-79)(Asimov, 382).

NUMBER ALPHABETS AND CODES

The idea that Bach had used what is called a "number alphabet" was the subject of four studies by Friedrich Smend, a theologian and musicologist. The first study was published in 1947, and has since been the subject of controversy and misapplication. At times, it would seem this idea has been taken to the point of absurdity. For example, a book written by Kees van Houten and Marinus Kasbergen makes the case (using wonderfully complicated conditions, e.g., converting all dates to the Rosicrucian calendar), that Bach had encoded into his music the date of his own death! This statement is based on coded information presumably contained in the fifteen three-part *Sinfoniae* (1727) and the *Magnificat* (1730) (Tatlow, 1).

Not content with that, they go on to point out they have devoted a "special chapter" in their

book to the number of days Bach was alive, 23,869, also imbedded in his work. Upon reading this number, the following relationship will of course lead out at the reader: if one takes the number 23,869 and multiplies all of the digits together, $2 \times 3 \times 8 \times 6 \times 9$, the result is 2592. Subtract 461, (a prime number), from 2592, then add the result to the original number. The end result is 26,000, which in years happens to be the length of time it takes the earth to complete a rotational precession! (Please forgive the cynicism, but if Bach could predict the length of his own life, then it would seem Ice Ages shouldn't be a problem). This type of manipulation of numbers and other attempts to read cosmic significance into every Aspect Of Bach seems to belong in the same category as "Elvis Lives" and "No New Taxes" (Kuhn, 190)(Tatlow, 1).

Putting aside the more extreme ideas, if the original theory of Bach's use of number systems or other mathematical relationships should prove to be true, it would certainly lead to a revolution in thought about music theory as practiced in the Late Baroque (Tatlow, 2).

Another angle on the question becomes apparent when Leo Schrade, referring to Bach, asks:

Was religious quality or intensity of feeling ... the real force that gave his work its form? ...we must learn to understand its significance in terms of his own situation and problems, or else it merely calls forth our subjective and uncontrolled imagination (Schrade, 4-5).

So, was the genius of Bach due only to some supernatural, magical quality of numbers, to the sterile perfection of mathematics? Or was it his personal devotion to his God and his church, or both, or neither? Author Philipp Spitta quotes Johann Mattheson (1681-1764) as saying that Bach had little training in the "deep theoretical study of music", (this statement would be looked upon with deep suspicion by any modern counterpoint class), and displayed no interest in using the "supposed mathematical bases of composition" in his teaching (Tatlow, 2).

If one were living in the Baroque, one of the most obvious places to use a number code would have been in figured bass. No indications have been found that Bach ever did this. Another might have been the use of cabalistic number alphabet codes, but even assuming that he knew of them, would he have taken the chance of being accused of using what was then considered to be magic, of weaving "spells" into his music? By all accounts, he was a Lutheran of some conviction, and would hardly have risked his reputation or position by following any such practice (Tatlow, 110, 126-8).

Probably the most well-known example of Bach's use of an underlying meaning in any of his music is the appearance of his name in what he had planned as the next-to-last fugue of *The Art of Fugue* (Kupferberg, 107).

Carl Philipp Emanuel Bach wrote:

In the course of this fugue, at the point where the name BACH was brought in as a countersubject, the composer died (Hofstadter, 81, 86).

Apparently, this was about as close as Bach would come to employing any deep mystical meaning in his music, outside of a religious reference. Ruth Tatlow has pointed out that much of Friedrich Smend's work is based upon faulty assumptions, and in some cases practically accuses Smend of outright negligence in his research. Another point to be considered: if Bach were indeed hiding secret messages in his music, to whom were they directed? It is commonly accepted that for the most part, Bach wrote only for himself, his employer, and his students. This does not, however, preclude the use of mathematics in other aspects of his work (Tatlow, 129)(Wolff, 806).

ARITHMETIC, ALGEBRAIC, AND GEOMETRIC CONSTRUCTIONS

In some works, there seems to be no shortage of veiled references. In Bach's *O Gott, du frommer Gott*, (Chorale #315), he shows his fondness for the number six, (which appears in several other of Bach's works), probably because of the number's importance in the Bible. This is a six-line chorale in G Major, the harmonization of which begins on the VI triad. There are six notes foreign to the key of G Major: four C sharps and two F naturals. Other "6" occurrences can be found: in leaps; voices that have no leaps; the ratio of repeated notes and leaps; and the occurrence of sixth chords (first inversions). [This reference went on to state that there were 18 "E"s (3x6), but I counted 20; nine in the treble staff and eleven in the bass. It also claimed six tied notes; I could count four, eight, or seven, (depending on one's definition), but not six] (Rubin, 340-343).

In *The Musical Offering* there is a three-voice canon labeled "Canon per Tonos" which has some interesting properties. The overall key of the canon is C minor, but by the time the ending is reached, the key has been unobtrusively shifted to D minor. The canon is so written as to permit a natural progression back to the beginning, and so end the second time in the key of E minor. If this is done a total of six times, the player will find the canon has returned to the original key of C minor, but all the voices are an octave higher (Hofstadter, 10).

A psychologist, Roger Shepard, has developed what are called Shepard-tone scales, in which ascending parallel scales are played in several octaves simultaneously. As the pitches rise, the loudness is gradually decreased in the top octaves and gradually increased in the bottom octave, with the apparent effect on the listener of continuously ascending scales (Hofstadter, 717-718).

If the aforementioned "Canon per Tonos" were played using the technique of Shepard-tone scales, the canon would seem to ascend to infinity. The mathematical version of this paradox was stated by mathematician Kurt Godel in his 1931 paper "On Formally Undecidable Propositions in *Principia Mathematica* and Related Systems I", the gist of which can be conveyed in an everyday verbal context by making the following statement: "This statement is false". Both Bach's canon and the nested levels of logic implied in Godel's Theorem are both paradoxes, in that they can go on seemingly forever, but never get anywhere. This is certainly an example of a mathematical pattern being applied in Bach's music, but did Bach consciously use what would become Godel's Theorem when writing this piece? Or was this internal relationship applied intuitively? (Personally, the latter seems much more probable than the former) (Hofstadter, 15-17).

Another portion of *The Musical Offering* is known as the "Crab Canon" which has the charming feature of using the same theme played against itself; one theme going forward, the other backward. In other words, it should sound the same played forward or backwards. This is, of course, the familiar Commutative Property: $(a+b)=(b+a)$. Not only can this relationship be found

in the canon, but the idea of "indirect self-reference" can be inferred. If these (rather convoluted) ideas have been followed properly by this somewhat perplexed music student, then "indirect self-reference" would seem to refer to more than one level of understanding. Perhaps a more lucid way to put it would be in the context of form and content. One can listen to the canon, and enjoy it, but still be ignorant of the message carried within the very form of the piece, i.e., its mirror-image quality. So, the form becomes part of the content, and clues to this fact may or may not be hidden within the content (or hidden within the form!) (Hofstadter, 199-230)(Keedy, 5-6).

Geometry was not foreign to Bach, either. Returning to his chorales, in the last three measures of *Jesu, meine Freude* (#138), a large triangle is delineated by two G-sharps in the bass and a G-natural in the treble. Other triangles are formed by other significant note combinations. This combination is only strange for Bach because usually employed the notes "B" and "C" to construct his triangles, many of them Golden Means (or Golden Sections) (Rubin, 344-349).

That Bach was no stranger to numbers can be illustrated by recalling a somewhat humorous incident in his life; when sent a cask of wine by his cousin, Johann Elias Bach, he took the time and trouble to calculate that the gift had actually cost him money, and wrote to his cousin asking that Elias not do him any similar favors in the future (Hindemith, 7). This simple exchange also points up his attention to detail, and a hint of a mischievous, (or perhaps abrasive), nature. It is not difficult to imagine such a person weaving pretty puzzles into his life's work, either for his own edification, or to elicit others' admiration (or consternation). If there are great compositional secrets hidden within this mathematical framework, they will likely remain hidden, since Bach himself left no clues on the subject. Magic, mysticism, and supernatural skills can also be relegated to a low probability. Rather, Bach's use of mathematics would seem to suggest a true love for detail in his work, perhaps an effort to utilize many aspects of human existence and knowledge, an effort to come to terms with our physical world, our perception of it and our role within it. Or, he may have just liked a good puzzle!

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